Running Coupling in the SU(2) Lattice Gauge Theory

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Abstract

Scenario according to which the SU(2)-gluodynamics is a theory with a nontrivial fixed point is analyzed from the point of view of the modern Monte-Carlo (MC) lattice data. It is found that an *assumption* of the first order fixed point $g = g_f$ of the beta function $\beta_f(g)$ has no contradictions with existing MC lattice data. The beta function parameters are found from the requirement of constant values for critical temperature T_c/Λ_L^{FP} and string tension $\sqrt{\sigma}/\Lambda_L^{FP}$ in MC lattice calculations at $4/g^2 \geq 2.30$.

The Monte–Carlo (MC) lattice simulations are one of the main sources of the non-perturbative results in the gauge field theories. For SU(N) gluodynamics on lattices of size $N_\tau \times N_\sigma^3$ MC results are the dimensionless functions of a bare coupling constant $\beta = 2N/g^2$. The transformation of these functions to physical quantities are done by multiplying them with a lattice spacing a in the corresponding powers. The length scale L ($V = L^3$ is the system volume) and the temperature T are given by

$$L \ = \ N_\sigma \ a \ , \quad T \ = \ (N_\tau \ a)^{-1}. \tag{1}$$

The lattices with $N_\sigma >> N_\tau$ correspond to the finite temperature models, whereas those with $N_\sigma \approx N_\tau$ are identified with zero temperature limit. Requirement $N_\tau >> 1$ is necessary to avoid lattice artifacts.

The best studied quantities in the MC simulations of pure $SU(N)$ gauge theories are the dimensionless string tension $(\sqrt{\sigma}a)_{MC}$ and the critical coupling β_c^{MC} of the deconfinement phase transition. The values of β_c^{MC} were found for the finite lattices and the extrapolation to spatially infinite volume ('thermodynamic limit') $N_\sigma \to \infty$ has been done (see Ref. [1] and references therein). In what follows we discuss $SU(2)$ gluodynamics. The MC values of the critical couplings β_c^{MC} for different N_τ and of $(\sqrt{\sigma}a)_{MC}$ for different β are presented in Table I and Table II, respectively. The data are taken from Ref. [1]. To define their physical values one needs a connection between the lattice spacing a and the bare coupling constant g. Such a connection is given in terms of the beta function $\beta_f(g)$ through the equation:

$$\beta_f(g) \ = \ - \ a\frac{dg}{da} \ . \tag{2}$$

The conventional perturbation theory gives the following expansion of the beta function

$$\beta_f^{AF}(g) \;=\; -\,b_0 g^3 - b_1 g^5 + O(g^7)\;, \qquad b_0 = \frac{11N}{48\pi^2}\;,\;\; b_1 = \frac{34}{3}\left(\frac{N}{16\pi^2}\right)^2\;. \tag{3}$$

N refers to the group $SU(N)$. Differential equation (2) with $\beta_f^{AF}(g)$ (3) leads to

$$a\Lambda_L^{AF} \;\cong\; \exp\left(-\frac{1}{2b_0 g^2}\right)\,(b_0 g^2)^{-b_1/2b_0^2} \;\equiv\; R(g^2) \tag{4}$$

where Λ_L^{AF} is an integration constant of Eq. (2). Eq. (4) is known as the asymptotic freedom (AF) relation.

Using Eqs. (1) and (4) one can calculate

$$T_c/\Lambda_L^{AF} \;\equiv\; \frac{1}{N_\tau a_c \Lambda_L^{AF}} \;=\; \frac{1}{N_\tau R(g_c^2)} \tag{5}$$

and

$$\sqrt{\sigma}/\Lambda_L^{AF} \;\equiv\; \frac{(\sqrt{\sigma}a)_{MC}}{a\Lambda_L^{AF}} \;=\; \frac{(\sqrt{\sigma}a)_{MC}}{R(g^2)}\;. \tag{6}$$

The values of T_c/Λ_L^{AF} (see also Ref. [1]) at different N_τ are presented in our Table I and of $\sqrt{\sigma}/\Lambda_L^{AF}$ for different couplings β in Table II. One observes a rather strong dependence of T_c/Λ_L^{AF} on N_τ and $\sqrt{\sigma}/\Lambda_L^{AF}$ on β. It means that the perturbative AF relation (4) does not work even on the largest available lattices. This fact is known as an absence of the asymptotic scaling. In contrast to the problem with an asymptotic scaling the scaling has been observed for the ratios of different physical quantities calculated from the lattice expectation values. MC data from Tables I and II give for different lattices almost a constant ratio [1]:

$$\left(\frac{T_c}{\sqrt{\sigma}}\right)_{MC} \;=\; 0.69\;\pm\;0.02 \tag{7}$$

if MC values for T_c/Λ_L^{AF} (5) and $\sqrt{\sigma}/\Lambda_L^{AF}$ (6) are calculated at equal coupling constants β in the region $\beta \geq 2.30$. It suggests a possibility of the universal asymptotic scaling violation: it has been proposed in Ref. [2] that a deviation from the asymptotic scaling can be described by a universal 'non-perturbative' (NP) beta function, i.e., $\beta_f^{NP}(g)$ is the same for all lattice observables and it does not depend on the lattice size if N_σ and N_τ are not too small.

The following ansatz was suggested [2]:

$$a\Lambda_L^{NP} \;=\; \lambda(g^2)R(g^2)\;, \tag{8}$$

where $R(g^2)$ is given by Eq. (4) and $\lambda(g^2)$ is thought to describe a deviation from the perturbative behaviour. The equation (4) has been expected at $g \to 0$ so that an additional constraint, $\lambda(0) = 1$, has been assumed. The values of T_c/Λ_L^{NP} and $\sqrt{\sigma}/\Lambda_L^{NP}$ can be calculated then as

$$T_c/\Lambda_L^{NP} \;=\; \frac{1}{N_\tau\;\lambda(g_c^2)R(g_c^2)}\;, \tag{9}$$

$$\sqrt{\sigma}/\Lambda_L^{NP} \;=\; \frac{(\sqrt{\sigma}a)_{MC}}{\lambda(g^2)R(g^2)}\;. \tag{10}$$

A simple formula for the function $\lambda(g^2)$ was suggested [2]:

$$\lambda(g^2) \;=\; \exp\left(\frac{c_3 g^6}{2b_0^2}\right) \; . \tag{11}$$

Parameter c_3 in Eq. (11) and a new one, $T_c^*/\Lambda_L^{NP} = const$, were considered as free parameters and determined from fitting the MC values of T_c/Λ_L^{NP} (9) at different N_τ to the constant value T_c^*/Λ_L^{NP}. This procedure gives:

$$T_c^*/\Lambda_L^{NP} \;=\; 21.45(14) \; , \qquad c_3 \;=\; 5.529(63)\cdot 10^{-4} \; . \tag{12}$$

The numerical values of T_c/Λ_L^{NP} (9) are presented in our Table I. In comparison to T_c/Λ_L^{AF} one observes much weaker N_τ dependence of T_c/Λ_L^{NP} (9). They become now close to the constant value T_c^*/Λ_L^{NP} (12). Due to Eq. (7) the constancy of T_c/Λ_L^{NP} (9) guarantees an approximate constancy of the physical string tension $\sqrt{\sigma}/\Lambda_L^{NP}$ (10) with the average value $\sqrt{\sigma^*}/\Lambda_L^{NP} = 31.56$ in the region of coupling constant $\beta = 2.3 \div 2.8$ (see Table II).

In spite of the evident phenomenological success of the above procedure of Ref. [2] the crucial question regarding the validity of the perturbative AF relation (4) at $g \to 0$ is not solved and remains just a postulate. Do the existing MC data rule out any other possibility? To answer this question we reanalyze the same MC data using the same strategy as in Ref. [2]. A principal difference of our analysis is that we do not assume the AF relation (4) between g and a at $g \to 0$. Instead of this standard approach we check a quite different scenario with the fixed point (FP) g_f of the beta function. Note that the so-called FP field theory models were considered a long time ago [3]. This theoretical possibility has been also discussed in Ref. [4]. It was demonstrated in Ref. [5] that the precise data on deep inelastic scattering do not eliminate the FP model and other tests would be necessary to distinguish between AF and FP QCD.

Let us *assume* that the beta function of the SU(2)-gluodynamics has a zero of the first order at some FP g_f. We thus have in the vicinity of this point

$$\beta_f^{FP}(g) \;=\; -\,b\,(g \;-\; g_f) \; . \tag{13}$$

From Eqs. (2) and (13) we find then

$$a\,\Lambda_L^{FP} \;=\; (g \;-\; g_f)^{1/b} \; , \tag{14}$$

with Λ_L^{FP} being an arbitrary integration constant of differential equation (2). It follows then for the critical temperature

$$T_c/\Lambda_L^{FP} \;=\; \frac{1}{N_\tau (g_c \;-\; g_f)^{1/b}} \tag{15}$$

and for the string tension

$$\sqrt{\sigma}/\Lambda_L^{FP} \;=\; \frac{(\sqrt{\sigma}a)_{MC}}{(g \;-\; g_f)^{1/b}} \; . \tag{16}$$

Our requirement similar to that of Ref. [2] is to fit T_c/Λ_L^{FP} (15) for different N_τ to a constant:

$$T_c^*/\Lambda_L^{FP} \equiv C_T = const \ , \tag{17}$$

where numerical value of C_T is a priori unknown. This requirement leads to the following expression for the critical coupling

$$g_c = (C_T N_\tau)^{-b} + g_f \ , \quad \beta_c = \frac{4}{[(C_T N_\tau)^{-b} + g_f]^2} \ . \tag{18}$$

In addition we require the constancy of $\sqrt{\sigma}/\Lambda_L^{FP}$ (16) for different β. It introduces another unknown constant

$$\sqrt{\sigma^*}/\Lambda_L^{FP} \equiv C_\sigma = const \tag{19}$$

and leads to the model equation for $\sqrt{\sigma}a$ as a function of β:

$$\sqrt{\sigma}a = C_\sigma (g - g_f)^{1/b} \equiv C_\sigma (2/\sqrt{\beta} - g_f)^{1/b} \ . \tag{20}$$

Our fitting procedure for finding beta function parameters g_f and b as well as the constants C_T (17) and C_σ (19) is to minimize χ^2 defined as

$$\chi^2 = \sum_{N_\tau} \frac{\left[(\beta_c - \beta_c^{MC})\right]^2}{[\Delta\beta_c^{MC}]^2} + \sum_{\beta_i} \frac{[\sqrt{\sigma}a - (\sqrt{\sigma}a)_{MC}]^2}{[\Delta(\sqrt{\sigma}a)_{MC}]^2} \ , \tag{21}$$

where β_c and $\sqrt{\sigma}a$ are given by model equations (18) and (20), $\Delta\beta_c^{MC}$ and $\Delta(\sqrt{\sigma}a)_{MC}$ stand for uncertainties of the corresponding MC data. We assume that deviations from the formula (14) are negligible for $\beta \geq 2.30$ and use available MC data from Tables I and II satisfying this criterion: in the first sum of Eq. (21) we include MC points β_c^{MC} from Table I for $N_\tau = 5, \ 6, \ 8, \ 16$ and the second sum is carried over $\beta_i = 2.3, \ 2.4, \ 2.5, \ 2.6, \ 2.7, \ 2.85$ with corresponding MC values of $(\sqrt{\sigma}a)_{MC}$ from Table II.

The χ^2 reaches its minimum $\chi^2_{min} = 2.15$ at

$$g_f = 0.563 \ , \quad b = 0.111 \ , \quad C_T = 3.15 \ , \quad C_\sigma = 4.59 \ . \tag{22}$$

For the set of parameters (22) the values of T_c/Λ_L^{FP} (15) and $\sqrt{\sigma}/\Lambda_L^{FP}$ are presented in Tables I and II. They are constant within the errors[1] for different lattice sizes and different β values for $\beta \geq 2.30$. In Figs. 1 and 2 our fits for β_c (18) and $(\sqrt{\sigma}a)$ (20) (with the same parameter set (22)) are compared with MC values of β_c^{MC} and $(\sqrt{\sigma}a)_{MC}$ from Tables I and II.

The standard criterion $\chi^2 < \chi^2_{min} + 1$ [6] defines an confidence region of the model parameters which gives a rather large variance of g_f, b, C_T, C_σ, but with a very strong correlation between them. In Fig. 3 we show the projection of this confidence region to the (g_f, b) plane. At the end point A, $(g_f = 0.288, b = 0.800)$, we find $C_T = 0.174$ and $C_\sigma = 0.253$. Another end point B, $(g_f = 0.717, b = 0.142)$, corresponds to $C_T = 8.90$ and

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[1] The errors are induced solely by the uncertainties of the MC data for β_c^{MC} and $(\sqrt{\sigma}a)_{MC}$ in each point, and do not include the variance of the fit parameters.

$C_\sigma = 12.98$. Physical observables stay almost unchanged under large variance of the model parameters due to their strong correlations. We find

$$T_c/\sqrt{\sigma} \;\equiv\; C_T/C_\sigma \;=\; 0.687 \pm 0.005 \;, \tag{23}$$

where the error in Eq. (23) caused by the parameter variations is calculated from the variance matrix of the parameters [6].

From Eq. (8) one can easily reconstruct the NP beta function $\beta_f^{NP}(g)$

$$\beta_f^{NP} \;=\; -\;\frac{b_0^2 g^3}{b_0 - b_1 g^2 + 3 c_3 g^8}\;. \tag{24}$$

In Fig. 4 we compare different beta functions discussed in our paper: $\beta_f^{AF}(g)$ (3), $\beta_f^{NP}(g)$ (24) and $\beta_f^{FP}(g)$ (13). It is amazing that $\beta_f^{NP}(g)$ with NP corrections (8) of Ref. [2] is very close to our straight line $\beta_f^{FP}(g)$ (13) in a rather wide region of the coupling constant. Note also that different numerical values of T_c^*/Λ_L^{NP} and T_c^*/Λ_L^{FP} are caused by a freedom in choosing the form of expression (14) defining the arbitrary integration constant of differential equation (2).

At the present moment there are no SU(2) MC data for the critical temperature and the string tension in the region $\beta > 2.85$. In the framework of our FP scenario we can predict the values of $\sqrt{\sigma}a$ and β_c for the future MC calculations. They are presented in Tables III and IV. As is seen from Figs. 1 and 2 these numbers can be hardly distinguished from those obtained with NP corrections (8) [2] to the AF relation. One needs very large values of β to observe the difference. The principal difference is of course exist: $\beta_c \to \infty$ for $N_\tau \to \infty$ in the approach of Ref. [2] and $\beta_c \to 4/g_f^2 \;=\; const$ for $N_\tau \to \infty$ in the FP scenario.

It is clear that obtaining MC data for the critical temperature and string tension in the $SU(2)$ gauge theory for the region of β essentially greater than 2.85 would require very large lattices and hardly possible in the nearest future. It seems that these restrictions are not so severe for the finite volume observables in the SU(2) gauge theory. In Ref. [7] the quantity $\bar{g}^2(L)$ defined as the response of the system in hypercube $L \times L \times L \times L$ to a constant color-electric background field was studied for $\beta = 2.6 \div 3.7$. To check the consistency of our FP scenario with these results we reanalyzed some MC data obtained in Ref. [7]. Namely we use in our analysis the sets of the bare couplings β tuned to achieve constant values of $\bar{g}^2(L_j)$, $(j = 0, 2, 4, 6, 8)$ [2] for different lattice sizes $N_\sigma = N_\tau \equiv N$.

From Eq. (14) it follows that the dependence of g on N at constant $\bar{g}^2(L_j)$ is given by

$$g_j(N) \;=\; \left(\frac{L_j \Lambda^{FP}}{N}\right)^b + g_f \;, \tag{25}$$

where relation $a = L/N$ has been used. In contrast to our previous consideration, the cutoff dependence (lattice artifact) is not negligible in the present case. Following to Ref. [7] we assume that it is proportional to $1/N$. Therefore to fit the data from the Table V we use the following formula

[2] We enumerate L_j with successive even numbers to retain the notations of Ref. [7].

$$\beta_j(N) = \frac{4}{(g_j(N))^2} - \frac{C_j}{N}, \tag{26}$$

where C_j is a constant.

In our fit procedure the FP beta function parameters g_f and b have been fixed at their values (22) found from the previous analysis, and $L_j\Lambda^{FP}$ and C_j for each column of Table V are chosen to minimize the following expression

$$\chi_j^2 \ = \ \sum_N \frac{[\beta_j - \beta_j^{MC}]^2}{[\Delta\beta_j^{MC}]^2}. \tag{27}$$

The results of the fit are shown in Table VI and in Fig. 5.

For a comparison we have made the similar fit assuming AF behavior of $g_j(N)$, i.e. instead of Eq. (25) the value of $g_j(N)$ was determined as a solution of the transcendental equation

$$\frac{L_j\Lambda^{AF}}{N} = R\left(g_j^2(N)\right) \tag{28}$$

with function R defined by Eq. (4). As is seen from Table VI, χ_8^2/dof is almost two times smaller in the FP scenario then that in AF one. This value correspond to the region $1.17 \leq g \leq 1.25$, where the difference between AF and FP beta-functions becomes large (See Fig. 4). For the rest of the data sets both approaches give nearly the same fit quality. Still, the FP scenario provides a better agreement of the ratios L_j/L_8 with those of Ref. [7] shown in the last column of Table VI. Therefore, MC data of Ref. [7] are consistent with FP beta function behavior (13,22). In the region of $g \approx 1$ we do not observe the difference between AF and FP scenarios: as seen from Fig. 4 β_f^{AF} and β_f^{FP} are close to each other in this region of g. Hopefully additional lattice calculations in the spirit of Ref. [7] at bare coupling $g < 1$ would allow one to find the true zero of the beta function $\beta_f(g)$.

We conclude that available MC lattice data in the SU(2)-gluodynamics do not exclude the possibility of the FP scenario (14). New lattice data are necessary to prove (or disprove) the AF relation (4).

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TABLES

TABLE I. MC data for critical couplings β_c^{MC} at different N_τ are taken from Ref. [1]. The values of T_c/Λ_L^{AF} are calculated from Eq. (5) assumed the perturbative AF relation (4) (see also Ref. [1]). T_c/Λ_L^{NP} are obtained from Eq. (9). Our results for T_c/Λ_L^{FP} followed from Eq. (15) in the FP scenario are presented in the last column.

N_τ	β_c^{MC}	T_c/Λ_L^{AF}	T_c/Λ_L^{NP}	T_c/Λ_L^{FP}
2	1.8800(30)	29.7	8.65(12)	1.349(16)
3	2.1768(30)	41.4	18.69(21)	2.696(29)
4	2.2988(6)	42.1	21.44(05)	3.084 (7)
5	2.3726(45)	40.6	21.95(33)	3.167(48)
6	2.4265(30)	38.7	21.81(22)	3.156(32)
8	2.5115(40)	36.0	21.44(27)	3.124(41)
16	2.7395(100)	32.0	21.50(64)	3.200(99)

TABLE II. MC data of $(\sqrt{\sigma}a)_{MC}$ for different lattices and coupling constants are taken from Ref. [1]. The values $\sqrt{\sigma}/\Lambda_L^{AF}$ and $\sqrt{\sigma}/\Lambda_L^{NP}$ are calculated from Eqs.(6) and (10), respectively. Last column corresponds to our results for $\sqrt{\sigma}/\Lambda_L^{FP}$ (16) in the FP scenario.

N_σ	N_τ	β	$(\sqrt{\sigma}a)_{MC}$	$\sqrt{\sigma}/\Lambda_L^{AF}$	$\sqrt{\sigma}/\Lambda_L^{NP}$	$\sqrt{\sigma}/\Lambda_L^{FP}$
8	10	2.20	0.4690(100)	61.7(14)	28.56 (61)	4.116(88)
10	10	2.30	0.3690(30)	62.4(5)	31.78 (26)	4.574(38)
16	16	2.40	0.2660(20)	57.8(4)	31.94 (25)	4.615(35)
32	32	2.50	0.1905(8)	53.3(2)	31.51 (14)	4.587(20)
20	20	2.60	0.1360(40)	49.0(14)	30.70 (91)	4.509(133)
32	32	2.70	0.1015(10)	47.1(5)	31.03 (31)	4.601(46)
48	56	2.85	0.0630(30)	42.8(21)	30.00(143)	4.511(215)

TABLE III. Predictions for $(\sqrt{\sigma}a)_{MC}$ at different β according to Eq. (20).

β	2.90	2.95	3.00	3.05	3.10
$\sqrt{\sigma}a$	0.0552(14)	0.0476(15)	0.0411(16)	0.03555(17)	0.0308(17)

TABLE IV. Predictions for β_c^{MC} at different N_τ according to Eq. (18).

N_τ	20	24	28	32
β_c	2.8077(49)	2.8683(71)	2.9201(93)	2.9653(115)

TABLE V. The bare coupling β at different lattice size N for fixed $\bar{g}^2(L)$ [7]. The uncertainties of β in columns 2–5 were recalculated from errors of $\bar{g}^2(L)$, given in Ref. [7], using linear interpolation for the dependence $\bar{g}^2(L)$ on β at fixed N.

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N	β				
	$\bar{g}^2(L_0) = 2.037$	$\bar{g}^2(L_2) = 2.380$	$\bar{g}^2(L_4) = 2.840$	$\bar{g}^2(L_6) = 3.550$	$\bar{g}^2(L_8) = 4.765$
5	3.4564(25)	3.1898(22)	2.9568(19)	2.7124(34)	
6	3.5408(40)	3.2751(35)	3.0379(31)	2.7938(48)	2.5752(28)
7	3.6045(42)	3.3428(36)	3.0961(33)	2.8598(50)	2.6376(20)
8	3.6566(47)	3.4009(42)	3.1564(38)	2.9115(55)	2.6957(21)
10	3.7425(59)	3.5000(61)	3.2433(53)	3.0071(77)	2.7824(22)
12					2.8485(32)
14					2.9102(62)

TABLE VI. The results of fitting the data from Table V with Eq. (26) assuming FP (25) and AF (28) scenario. The ratios L_j/L_8 are compared with those calculated in Ref. [7]. The number of degrees of freedom (dof), the difference between the number of points and the number of fit parameters, equals 3 for $j = 0, 2, 4, 6$ and 4 for $j = 8$.

	FP				AF				Ref. [7]
j	χ_j^2/dof	C_j	$L_j\Lambda_L^{FP} \times 10$	L_j/L_8	χ_j^2/dof	C_j	$L_j\Lambda_L^{AF} \times 10^2$	L_j/L_8	L_j/L_8
0	1.25	0.315	0.102	0.072	1.08	0.215	0.139	0.080	0.070(8)
2	0.09	0.559	0.187	0.131	0.09	0.378	0.254	0.146	0.124(13)
4	0.75	0.422	0.387	0.272	0.81	0.177	0.512	0.293	0.249(19)
6	0.14	0.585	0.734	0.516	0.18	0.220	0.933	0.534	0.500(23)
8	0.89	0.589	1.442	1.000	1.67	0.032	1.742	1.000	1.000

FIGURES

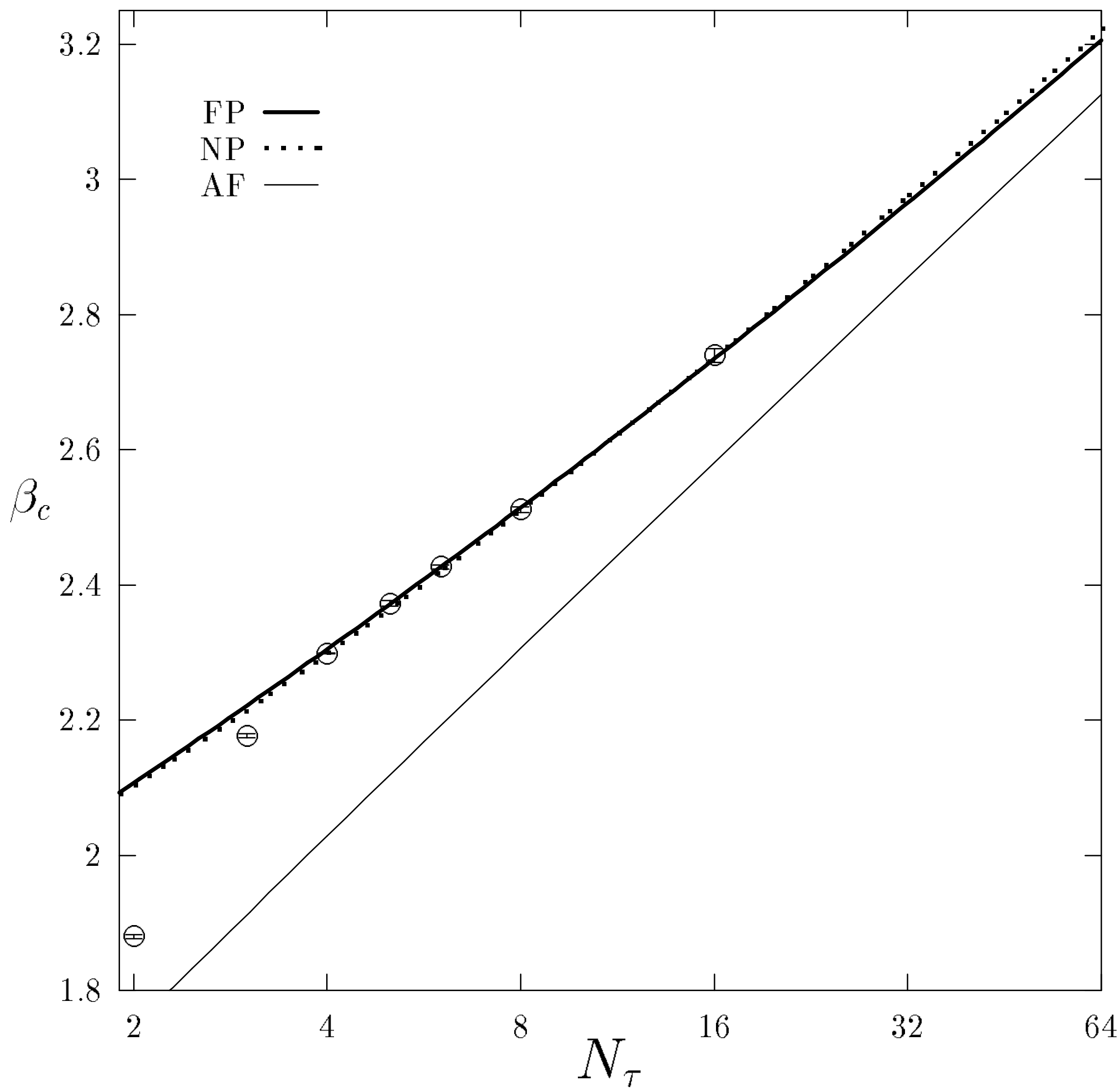

FIG. 1. Circles with errorbars represent the MC data listed in Table I. The bold solid line corresponds to Eq. (18) of the FP scenario. The dotted and thin solid lines show $\beta_c = 4/g_c^2$ found by solving the equations $T_c^*/\Lambda_L^{NP} = [N_\tau \lambda(g_c^2) R(g_c^2)]^{-1}$ and $T_c^*/\Lambda_L^{NP} = [N_\tau R(g_c^2)]^{-1}$, respectively, with $T_c^*/\Lambda_L^{NP} = 21.45$.

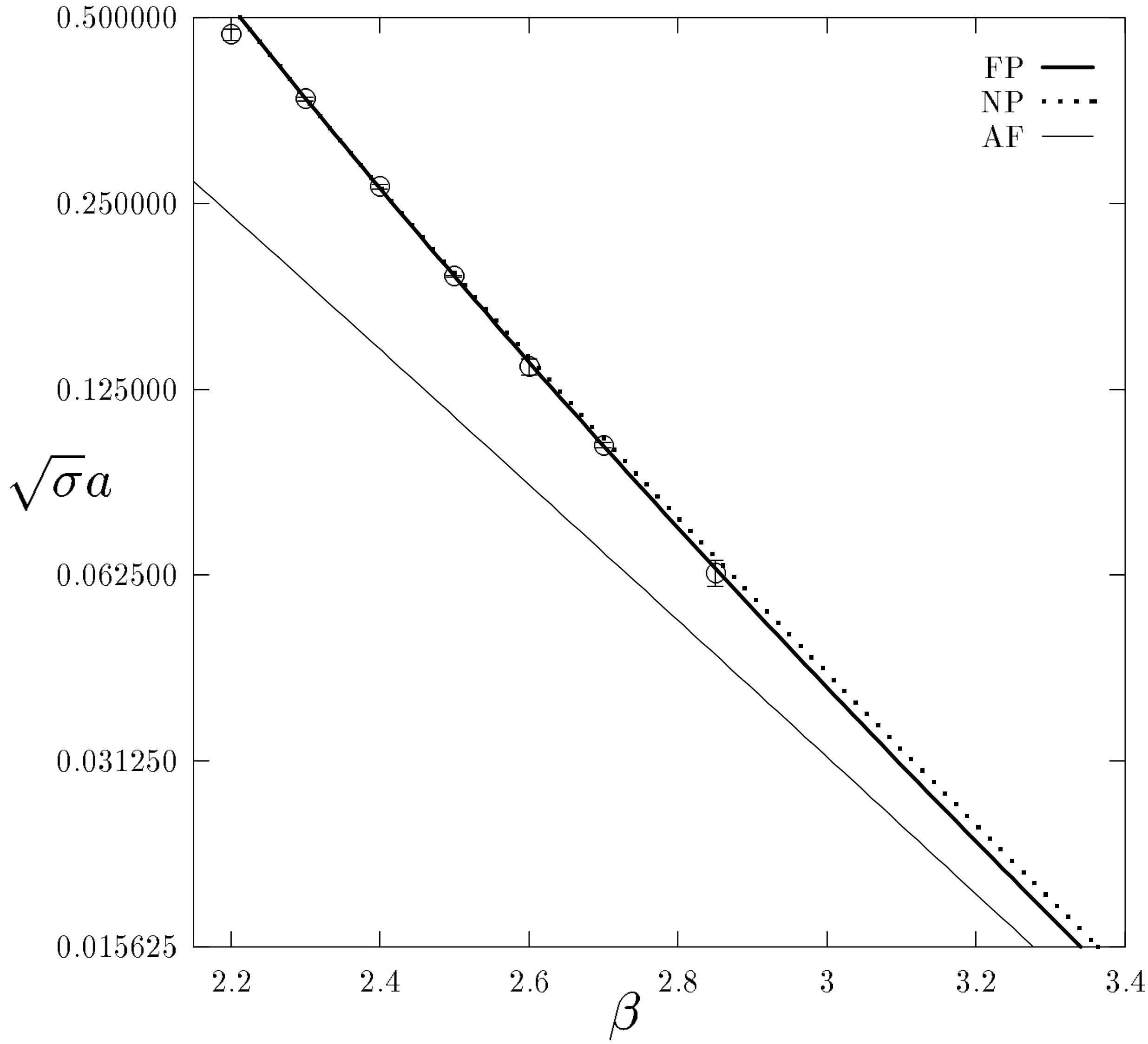

FIG. 2. Circles with errorbars represent the MC data listed in Table II. The bold solid line corresponds to Eq. (20) of the FP scenario. The dotted and thin solid lines are calculated as $\lambda(g^2)R(g^2)\sqrt{\sigma^*}/\Lambda_L^{NP}$ and $R(g^2)\sqrt{\sigma^*}/\Lambda_L^{NP}$, respectively, with $\sqrt{\sigma^*}/\Lambda_L^{NP} = 31.56$.

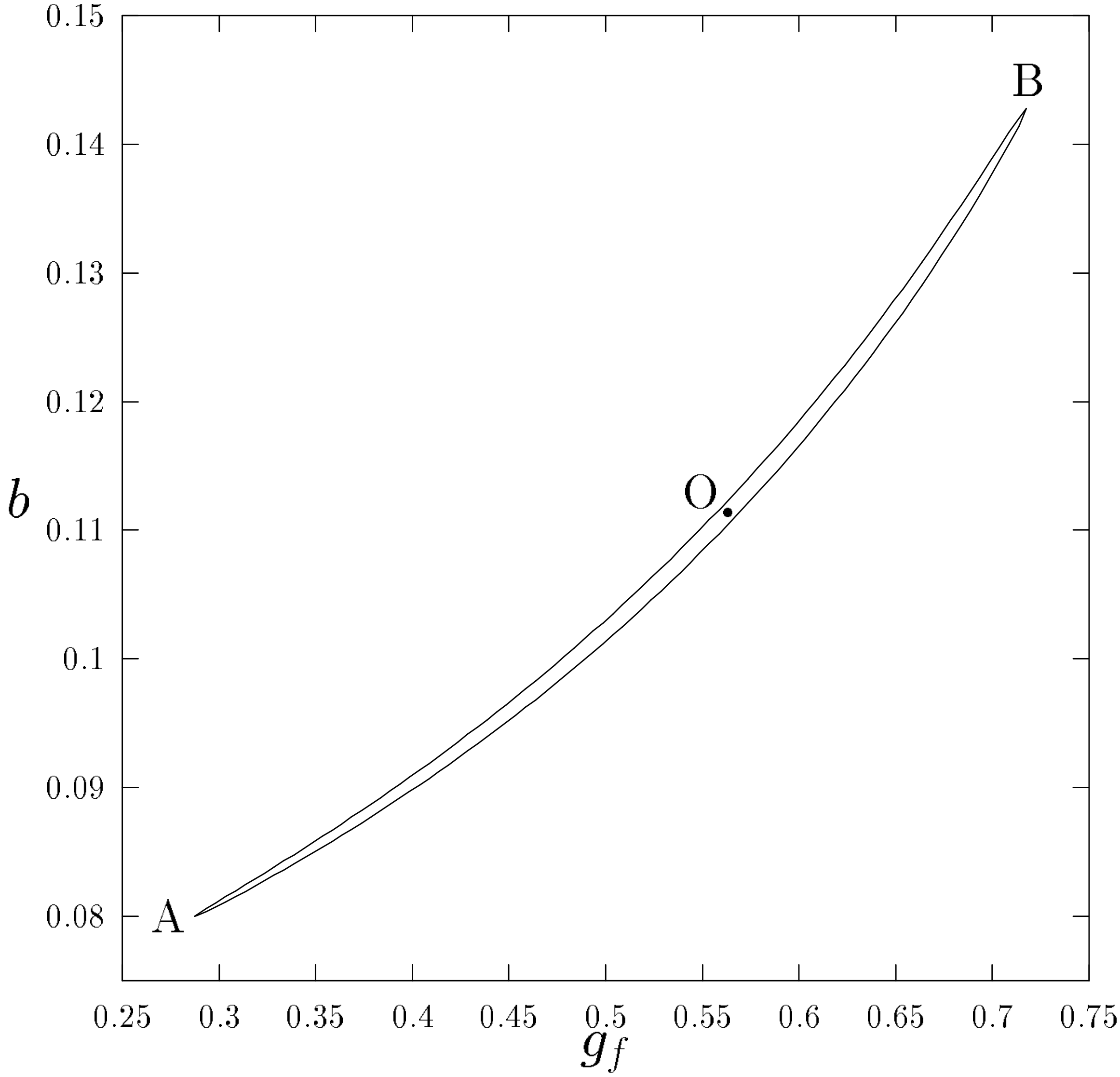

FIG. 3. Projection of the confidence region of the parameters to the (g_f, b) plane. The point O corresponds to the best fit (22).

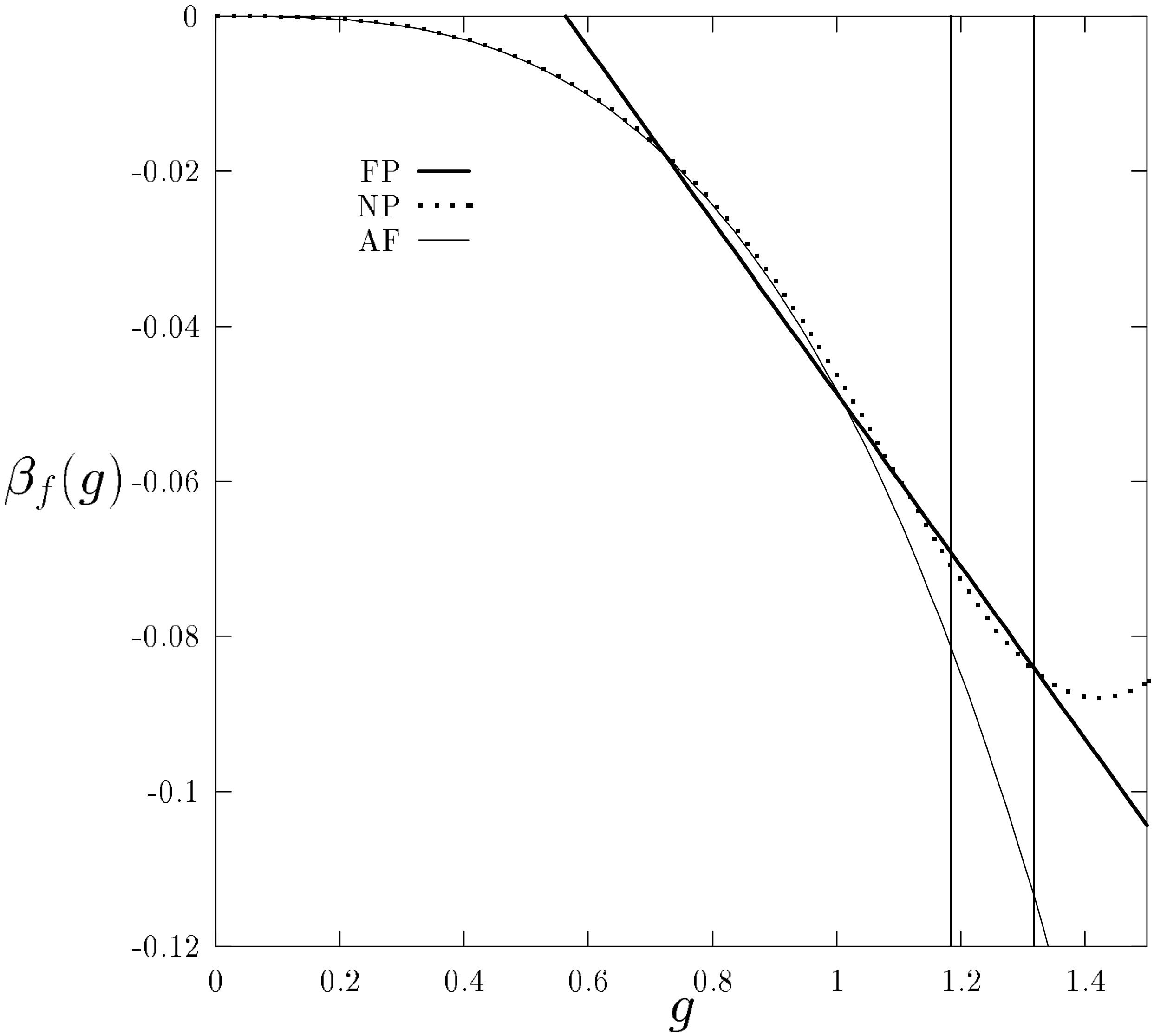

FIG. 4. The behavior of $\beta_f^{FP}(g)$ (13) (bold solid line), $\beta_f^{NP}(g)$ (24) (dotted line) and $\beta_f^{AF}(g)$ (3) (thin solid line). Two vertical lines show the region of the coupling constant ($\beta = 2.3 \div 2.8$) of MC data used in the fitting procedures.

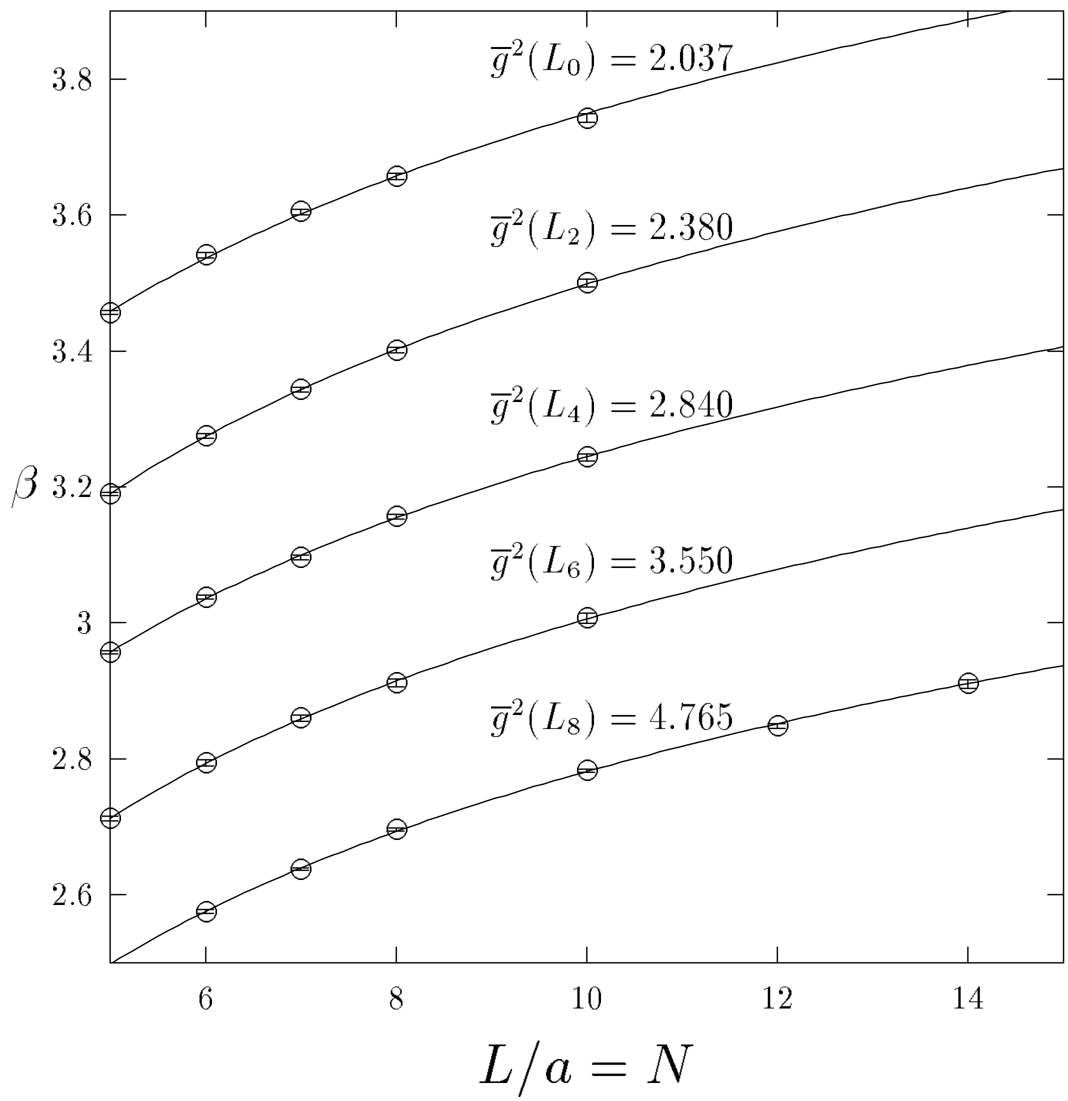

FIG. 5. The results of fitting the data from Table V with the formulas (25) and (26).